Broadening of spectral lines

An individual atom/molecule making a transition between energy levels emits one photon with a well-defined energy / frequency.

However, profiles of real spectral lines are not infinitely narrow.

e.g. for an emission line, width of the spectral line $\Delta \nu$ could be defined as the full width at half the maximum intensity of the line.

Details of definition don’t matter - important to see what causes lines to have finite width.
Two basic mechanisms:

1) Energy levels themselves are not infinitely sharp: emitted photons have a range of frequencies

2) Atoms and molecules in the gas are moving relative to the observer: observed photons don’t have the same frequency as the emitted photons because of the Doppler effect.
Consider excited state with energy $E$ above the ground state.

Electrons in excited state remain there for average time $\Delta t$ before decaying to ground state.

Uncertainty principle: energy of a level is uncertain by an amount $\Delta E$ given by:

$$\Delta E \Delta t \approx \frac{h}{2\pi}$$

But since $E = h\nu$, $\Delta E = h\Delta \nu$ 

$$\Delta \nu \approx \frac{1}{2\pi \Delta t}$$

Broadening due to this effect is called the natural linewidth.
Natural broadening

The line has a "Lorentz shape", sharper peak and wider wings than a Gaussian profile:

\[
\phi(v) = \frac{\Delta v / 2\pi}{(v - v_0)^2 + (\Delta v / 2\pi)^2}
\]

\[
\frac{1}{\Delta t} \approx A_{nn'}
\]

Where \(A_{nn'}\) is the A-Einstein coefficient of a transition between \(n \rightarrow n'\) and \(v_0\) is the mean frequency.
Natural linewidth sets absolute minimum width of spectral lines. However, normally very small - other effects dominate.

e.g. for hydrogen n=2 to n=1 transition (Lyman α transition, \( \nu = 2.5 \times 10^{15} \) Hz) the lifetime is of the order of \( 10^{-9} \) s.

Natural linewidth is \( \sim 10^8 \) Hz.

Compare to frequency of transition: \[ \frac{\Delta \nu}{\nu} \approx 10^{-8} \]

In astrophysical situations, other processes will often give much larger linewidths than this.
Collisional broadening

In a dense gas, atoms/molecules are colliding frequently. This effectively reduces the lifetime of states further, to a value smaller than the quantum mechanical lifetime.

If the frequency of collisions is $\nu_{\text{col}}$, then expect to get a collisional linewidth of about $\Delta \nu \sim \nu_{\text{col}}$.

Frequency of collisions increases with density - expect to see broader lines in high density regions as compared to low density ones.

e.g. a main sequence star (small radius) has a higher density at the photosphere than a giant of the same surface temperature. Spectral lines in the main sequence star will be broader than in the giant.
Collisional broadening

\[
\phi(\nu) = \frac{\Delta \nu_{tot} / 2\pi}{(\nu - \nu_0)^2 + (\Delta \nu_{tot} / 2\pi)^2}
\]

\[
\Delta \nu_{tot} = \Delta \nu + \frac{\nu_{col}}{\pi}
\]
Doppler or thermal broadening

Atoms/molecules in a gas have random motions that depend upon the temperature. For atoms of mass \( m \), at temperature \( T \), the typical speed is obtained by equating kinetic and thermal energy:

\[
\frac{1}{2}mv^2 = k_bT \quad \text{where } k_b = \text{Boltzmann's constant}
\]

Number of atoms with given velocity is given by Maxwell's law. Need to distinguish between forms of this law for speed and for any one velocity component:

\[
|\mathbf{v}|^2 = v_x^2 + v_y^2 + v_z^2
\]

Distribution of one component of the velocity, say \( v_x \), is relevant for thermal broadening - only care about motion along line of sight.
For one component, number of atoms $dN$ within velocity interval $dv_x$ is given by:

$$dN(v_x) \propto \exp\left(-\frac{mv_x^2}{2kT}\right)dv_x$$

Distribution law for speeds has extra factor of $v^2$:

$$dN(v) \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right)dv$$

Most probable speed:

$$v_{peak} = \sqrt{\frac{2kT}{m}}$$

Average speed:

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$$
Consider atom moving with velocity $v_x$ along the line of sight to the observer.

**Doppler shift** formula:  
\[ \frac{\nu - \nu_0}{\nu_0} = \frac{v_x}{c} \]

Combine this with the thermal distribution of velocities:

\[ \phi(\nu) = \frac{1}{\Delta \nu_D \sqrt{\pi}} \exp \left[ -\frac{(\nu - \nu_0)^2}{(\Delta \nu_D)^2} \right] \]

...where the **Doppler width** of the line:

\[ \Delta \nu_D = \frac{\nu_0}{c} \sqrt{\frac{2kT}{m}} \]
If the gas also has large-scale (i.e. not microscopic) motions due to turbulence, those add to the width:

\[
\Delta \nu_D = \frac{\nu_0}{c} \left( \frac{2kT}{m} + \nu_{turb}^2 \right)^{1/2}
\]

\(\nu_{turb}\) is a measure of the typical turbulent velocity (note: really need same velocity distribution for this to be strictly valid).

Some numbers for hydrogen:

\[
\frac{\Delta \nu_D}{\nu_0} \approx 4.3 \times 10^{-5} \left( \frac{T}{10^4 \text{ K}} \right)^{1/2}
\]

\[
\frac{\Delta \nu_D}{\nu_0} \approx 13 \left( \frac{T}{10^4 \text{ K}} \right)^{1/2} \text{ km s}^{-1}
\]

larger than natural linewidth

measured in velocity units, comparable to the sound speed in the gas
Doppler / thermal broadening important in many situations, e.g. in the spectra of stars. Turbulent component only matters when the temperature is very low / velocity high.

Closest massive star forming region turbulence in gas has been observed

Gas distribution in part of Orion: $T = 10 - 100$ K, $v \sim$ km/s
Thermal line profile

Voigt profile: combination of thermal and natural (or collisional) broadening

Gaussian: falls off very rapidly away from line center

Natural line profile falls off more slowly - dominates wings of strong lines
Summary:

**Width** of spectral lines depends upon:
- *Natural linewidth* (small)
- *Collisional linewidth* (larger at high density)
- *Thermal linewidth* (larger at higher temperature)
Absorption line and emission line spectra

Temperature of the Solar photosphere is ~6000K. Lots of spectral lines of different elements at this T.

Optical spectrum is an absorption line spectrum - see dark absorption lines superimposed on a bright continuum.

Small section of the Solar spectrum showing two strong lines due to sodium.
Absorption line spectra also seen in the spectra of distant quasars (light comes from gas flows around black holes):

Very large number of absorbers at different locations along the line of sight to the quasar.
Nebulae of different sorts typically show **emission line spectra**:

Spectral lines are **stronger** than the continuum.

Why this difference?
Use result we derived last time - consider radiation passing through a hot cloud of gas in thermal equilibrium:

Found:

\[ I_v(\tau_v) = I_0 e^{-\tau_v} + B_v (1 - e^{-\tau_v}) \]

Suppose no intensity entering the cloud, \( I_0 = 0 \). If the cloud is very optically thin:

\[ e^{-\tau_v} \approx 1 - \tau_v \]

\[ I_v(\tau_v) = B_v (1 - 1 + \tau_v) = \tau_v B_v \]
Optical depth is related to the absorption coefficient via:

\[ \tau_v = \alpha_v \Delta s \]  
(for constant \( \alpha \))

Means that:

\[ I_v = \tau_v B_v \propto \alpha_v B_v \]

Intensity is large at frequencies where the absorption coefficient is large.

For a hot gas, absorption coefficient is large at the frequencies of the spectral lines.

For an optically thin medium such as a nebula, expect an emission line spectrum with large intensity at the frequencies where \( \alpha_v \) is large.
Summary:

- **Strength** of different spectral lines depends upon the abundance of different elements, and on the excitation/ionization state (described in part by the Boltzmann formula).

- **Width** of spectral lines depends upon:
  - *Natural linewidth* (small)
  - *Collisional linewidth* (larger at high density)
  - *Thermal linewidth* (larger at higher temperature)

High quality spectrum gives information on composition, temperature and density of the gas.

c.f. `Modern Astrophysics` section 8.1: more on thermal broadening, Boltzmann law, and Saha equation (version of Boltzmann law for ionization).